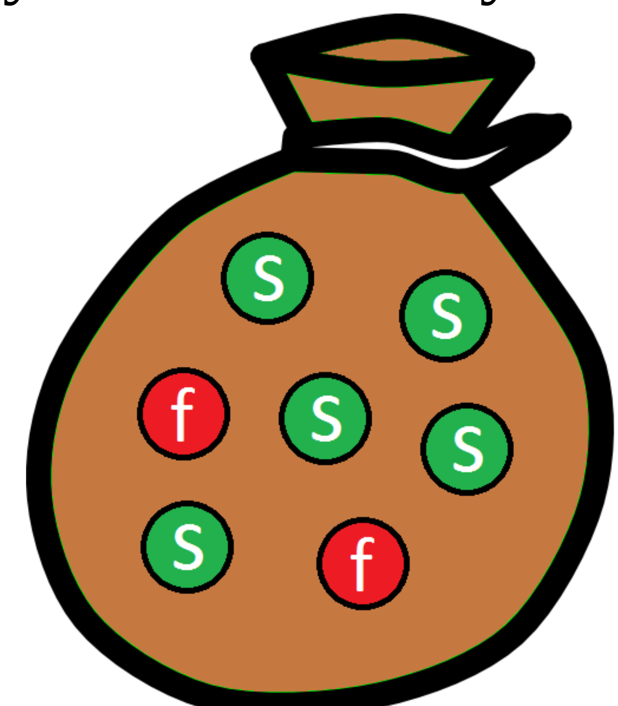
Discrete Random Variables
Day 2
Binomial Random Variables
Sec. 6.2

Preliminary Example of a Binomial Random Variable

Ex: A bag contains only 2 kinds of balls in it. Each ball in the bag has either the letter “s” (for success) or the letter “f” (for failure) on it. Suppose you draw 5 balls from the bag with replacement. Let X be the total number of successes you draw from the bag. What is the probability that you draw exactly 3 successes?



Preliminary Example of a Binomial Random Variable

<u>About Drawing Once</u>

Success = Drawing a green ball that has an "s" on it

Failure = Drawing a red ball that has an "f" on it

p = probability of success (when drawing 1 ball)

= 5/7 in this example

q = probability of failure (when drawing 1 ball)

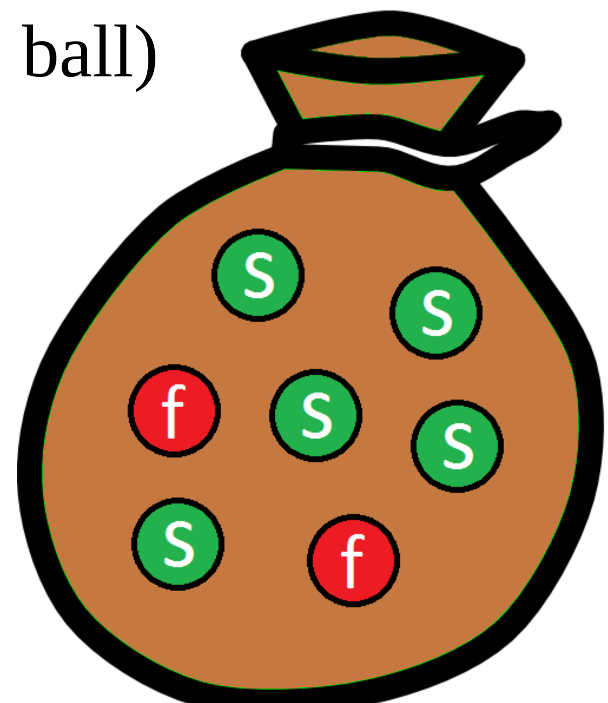
= 2/7 in this example

<u>About Drawing Five Times</u>

n = Total number of balls drawn

= 5 in this example

X = Total number of successes drawn



Preliminary Example of a Binomial Random Variable

$P(X = 3)$

$= P((f_1 \cap f_2 \cap s_3 \cap s_4 \cap s_5) \cup (f_1 \cap s_2 \cap f_3 \cap s_4 \cap s_5) \cup (f_1 \cap s_2 \cap s_3 \cap f_4 \cap s_5) \cup (f_1 \cap s_2 \cap s_3 \cap s_4 \cap f_5) \cup (s_1 \cap f_2 \cap f_3 \cap s_4 \cap s_5) \cup (s_1 \cap f_2 \cap s_3 \cap f_4 \cap s_5) \cup (s_1 \cap f_2 \cap s_3 \cap s_4 \cap f_5) \cup (s_1 \cap s_2 \cap f_3 \cap f_4 \cap s_5) \cup (s_1 \cap s_2 \cap f_3 \cap s_4 \cap f_5) \cup (s_1 \cap s_2 \cap s_3 \cap f_4 \cap f_5))$

$= P(f_1 \cap f_2 \cap s_3 \cap s_4 \cap s_5) + P(f_1 \cap s_2 \cap f_3 \cap s_4 \cap s_5) + P(f_1 \cap s_2 \cap s_3 \cap f_4 \cap s_5) + P(f_1 \cap s_2 \cap s_3 \cap s_4 \cap f_5) + P(s_1 \cap f_2 \cap f_3 \cap s_4 \cap s_5) + P(s_1 \cap f_2 \cap s_3 \cap f_4 \cap s_5) + P(s_1 \cap f_2 \cap s_3 \cap s_4 \cap f_5) + P(s_1 \cap s_2 \cap f_3 \cap f_4 \cap s_5) + P(s_1 \cap s_2 \cap f_3 \cap s_4 \cap f_5) + P(s_1 \cap s_2 \cap s_3 \cap f_4 \cap f_5)$

$= P(f_1) \cdot P(f_2) \cdot P(s_3) \cdot P(s_4) \cdot P(s_5) + P(f_1) \cdot P(s_2) \cdot P(f_3) \cdot P(s_4) \cdot P(s_5) + P(f_1) \cdot P(s_2) \cdot P(s_3) \cdot P(f_4) \cdot P(s_5) + P(f_1) \cdot P(s_2) \cdot P(s_3) \cdot P(s_4) \cdot P(f_5) + P(s_1) \cdot P(f_2) \cdot P(f_3) \cdot P(s_4) \cdot P(s_5) + P(s_1) \cdot P(f_2) \cdot P(s_3) \cdot P(f_4) \cdot P(s_5) + P(s_1) \cdot P(f_2) \cdot P(s_3) \cdot P(s_4) \cdot P(f_5) + P(s_1) \cdot P(s_2) \cdot P(f_3) \cdot P(f_4) \cdot P(s_5) + P(s_1) \cdot P(s_2) \cdot P(f_3) \cdot P(s_4) \cdot P(f_5) + P(s_1) \cdot P(s_2) \cdot P(s_3) \cdot P(f_4) \cdot P(f_5)$

Preliminary Example of a Binomial Random Variable

$P(X = 3)$

$= P(f_1) \cdot P(f_2) \cdot P(s_3) \cdot P(s_4) \cdot P(s_5) + P(f_1) \cdot P(s_2) \cdot P(f_3) \cdot P(s_4) \cdot P(s_5) +$
$P(f_1) \cdot P(s_2) \cdot P(s_3) \cdot P(f_4) \cdot P(s_5) + P(f_1) \cdot P(s_2) \cdot P(s_3) \cdot P(s_4) \cdot P(f_5) +$
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$P(s_1) \cdot P(f_2) \cdot P(s_3) \cdot P(s_4) \cdot P(f_5) + P(s_1) \cdot P(s_2) \cdot P(f_3) \cdot P(f_4) \cdot P(s_5) +$
$P(s_1) \cdot P(s_2) \cdot P(f_3) \cdot P(s_4) \cdot P(f_5) + P(s_1) \cdot P(s_2) \cdot P(s_3) \cdot P(f_4) \cdot P(f_5)$

$= q \cdot q \cdot p \cdot p \cdot p + q \cdot p \cdot q \cdot p \cdot p + q \cdot p \cdot p \cdot q \cdot p + q \cdot p \cdot p \cdot p \cdot q + p \cdot q \cdot q \cdot p \cdot p +$
$p \cdot q \cdot p \cdot q \cdot p + p \cdot q \cdot p \cdot p \cdot q + p \cdot p \cdot q \cdot q \cdot p + p \cdot p \cdot q \cdot p \cdot q + p \cdot p \cdot p \cdot q \cdot q$

$= p^3q^2 + p^3q^2 + p^3q^2 + p^3q^2 + p^3q^2 + p^3q^2 + p^3q^2 + p^3q^2 + p^3q^2 + p^3q^2$

$= 10p^3q^2$

$= {}_5C_3\ p^3q^2 \quad = \quad {}_5C_3 \left(\frac{5}{7}\right)^3 \left(\frac{2}{7}\right)^2 \quad = \quad 0.297495$

What is a Binomial Random Variable?

Situation

- A simple procedure is going to be performed a fixed amount of times (n times)
- Each time the simple procedure is performed, there is a notion of success and failure
- p is the probability of success if the simple procedure is performed once
- q is the probability of failure if the simple procedure is performed once
- $p+q=1$
- All runs (trials) of the simple procedure are independent of each other
- The random variable X counts the total number of successes obtained out of the n trials

What is a Binomial Random Variable?

Notes:

- There are 2 experiments going on here:
  - The simple procedure where something is performed once
  - The experiment we care about which consists of performing the simple experiment n times
- Success doesn’t always mean something positive. How do you know what a success is? It is whatever you are being asked to find the probability of

Formulas for a Binomial Random Variable

Probability Formula: $P(X = x) = {}_{n}C_{x}\, p^{x} q^{n-x}$

Expected Value Formula: $\mu = np$

Variance Formula: $\sigma^2 = npq$

Standard Deviation Formula: $\sigma = \sqrt{npq}$

Ex 1: Suppose you are going to draw 10 cards from a standard poker deck (with replacement).

a) What is the probability that exactly 7 of the cards are number cards?

b) What is the probability that you draw between 6 and 8 number cards inclusive?

c) What is the probability that you draw at least 8 number cards?

d) What is the probability that you draw at most 7 number cards?

e) Find the expected value, standard deviation and variance of the random variable in this problem.

f) Explain what each of the probabilities you calculated in parts (a)-(d) mean.

g) Explain the meaning of the expected value you calculated in part (e)

Ex 2 (Sec. 6.2 Ex 3 & 6 from the book): According to CTIA, 25% of all U.S. households are wireless-only households (no landline). In a random sample of 20 households, what is the probability that

a) Exactly 5 are wireless-only?

b) Fewer than 3 are wireless-only?

c) At least 3 are wireless-only?

d) The number of households that are wireless-only is between 5 and 7, inclusive?

e) Determine the mean, variance and standard deviation of the total number of wireless-only households (out of 20).

f) Explain what each of the probabilities you calculated in parts (a)-(d) mean.

g) Explain the meaning of the expected value you calculated in part (e)